Sequential negotiations with loss-averse buyers

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ABSTRACT

This paper analyzes sequential negotiations with exogenous breakdown risk between a risk-neutral seller and a loss-averse buyer who is privately informed about his valuation. I show that, compared to the risk-neutral benchmark, loss aversion on the buyer's side softens the rent-efficiency trade-off for the seller. The reason is that the higher the buyer's valuation is, the more he has to lose by rejecting the seller's offer. Thus, in equilibrium the seller's profits and overall efficiency are both higher than in the risk-neutral case. Moreover, I also show that loss aversion has a redistributive effect by increasing the equilibrium payoff of some low-valuation buyers and decreasing that of high-valuation ones.

"When you ain't got nothing, you got nothing to lose"

Bob Dylan, Like a Rolling Stone (1965)

1. Introduction

Many bargaining situations involve negotiations that take place over a period of time between parties that are differentially informed. Consider, for instance, a seller making a series of price offers to a buyer who is privately informed about his valuation. Three types of economic forces contribute to shape outcomes in such an environment. First, a buyer who has the option of deferring a purchase until later may have an incentive to wait for a better price offer even if his current payoff from making a purchase is positive. Second, a seller should anticipate strategic behavior by buyers when choosing prices. If a buyer is waiting for a better offer then the seller may have to set a price below the price she would set in a static setting, in order to induce the buyer to purchase earlier. Third, the possibility of making multiple price offers may allow the seller to engage in inter-temporal price discrimination and extract more surplus from buyers than what would be possible in a static setting. Hence, when the parties are differentially informed, delay serves as a signalling device for the informed party to communicate his private information, as well as a screening device for the uniformed party to gauge whether the other party is in a weak or strong bargaining position.

Moreover, many negotiations are inherently subject to various forms of breakdown risk. Maybe another buyer will arrive and offer the seller better terms of trade, or maybe the original buyer might discover another product that better suits his needs. For
example, suppose you are planning to buy a house and have started negotiating with one particular seller. Out of many factors that you may take into account during the negotiation, two important ones are: (1) How likely it is that if you wait for the seller to reduce the price, she will find another interested buyer and “disappear”? (2) How likely it is that if negotiations do breakdown you will find another house that matches your tastes? In fact, these risks in many situations are likely to be more important in evaluating the relative costs and benefits of delay than the standard discounting costs that play a crucial role in many bargaining models.

Sequential negotiations with asymmetric information have also been the focus of several laboratory experiments. By and large, however, equilibrium predictions from game theoretic models of sequential bargaining have fared poorly in these experiments. In particular, prices are often far higher than the levels predicted in models with risk-neutral buyers, and the buyers appear to be eager to make a purchase in earlier periods. Moreover, several comparative statics predictions about the effects of changes in information, changes in the discount factor, or changes in the time horizon are also rejected by the data. As some have argued, relaxing the assumption of risk-neutral preferences could potentially rationalize these experimental findings. Yet, while several studies have explored the role of risk preferences in static bargaining problems or in models of sequential bargaining with complete information, the role of risk preferences in models of sequential bargaining with asymmetric information has remained largely unexplored.

In this paper I introduce buyer loss aversion into a model of sequential negotiations with one-sided incomplete information and breakdown risk. I focus on loss aversion rather than classical risk aversion for several reasons. First, there is ample experimental and field evidence that people evaluate outcomes not (only) in absolute terms but (also) relative to a reference point, and that losses (relative to this reference point) loom larger than equal-size gains. Moreover, the recent literature in Behavioral Industrial Organization shows that in many buyer-seller interactions loss aversion seems to be empirically more relevant than risk aversion. For example, Heidhues and Köszegi (2014) show that buyer loss aversion is consistent with the puzzling combination of flexibility and stickiness observed in consumer prices; such a combination could not be rationalized by classical risk aversion alone. Similarly, most advertising and marketing techniques commonly employed by firms seem to be directly targeted at creating an “attachment effect” (Köszegi and Rabin, 2006) and exploiting consumers’ feeling of loss from failing to buy a product they were expecting to obtain. Most of the papers in this literature, however, impose (some or all of) the following assumptions: (i) the seller can commit in advance to a specific pricing strategy; (ii) consumers have homogenous preferences and no private information, and (iii) the interaction between the seller and the consumers lasts only one period. Yet, many buyer-seller interactions take place in a dynamic environment where the seller is unable to commit in advance to a given price. Moreover, consumer loss aversion is likely to play a relevant role in dynamic negotiations as consumers do not know what the future terms of trade will be if they refuse to buy at the current price.

Following Köszegi and Rabin (2006), I assume that the buyer’s reference point is given by his rational expectations, which are determined endogenously in the model by requiring that they must be consistent with optimal behavior given expectations. Since in bargaining negotiations parties cannot credibly commit in advance to a given strategy, the expectations about the possible outcomes of the bargaining process with which a party enters the negotiation may play an important role in assessing the outcome of the negotiation itself. To what extent then do the expectations with which parties enter a negotiation matter? I show that, compared to the risk-neutral case, loss aversion with expectations as the reference point actually makes delay and disagreement less likely, thus increasing overall efficiency. The intuition behind this result — which may appear counterintuitive at first — is as follows. In equilibrium, a buyer with a high valuation expects to achieve a large surplus and since expectations are the reference point, he suffers a large loss when delaying (or failing to reach) agreement. In other words, a high-value-loss-averse buyer has a lot to lose and so he is more willing to make concessions and to accept a high price. Hence, loss aversion with expectations as the reference point softens the rent-efficiency trade-off for the seller.

Section 2 introduces the model and the buyer’s preferences, and describes the solution concept. I focus on two-period sequential negotiations between a risk-neutral seller and a loss-averse buyer who is privately informed about his valuation (i.e., one-sided private information). In each period the seller makes a take-it-or-leave-it (TITO) offer to the buyer who can either accept or refuse and at the end of the first period there is an exogenous probability of breakdown. The assumption about the seller being risk neutral and having all the bargaining power is appropriate if we think of the seller as a large, profit-maximizing firm with (some) market power. Furthermore, this assumption is rather common in the current literature in Behavioral IO with loss-averse consumers.

Section 3 analyzes the case where the buyer’s valuation can take one of two values (high or low) and characterizes the unique equilibrium of the model. Moreover, I show that for some values of the seller’s prior beliefs about the buyer’s valuation, in equilibrium the buyer must play a mixed strategy. Delay still happens with positive probability, but it is less likely to arise when the

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2 On the role of risk aversion in models of bargaining with incomplete information see, for instance, Chae and Heidhues (2004), Roth (1985) and Volij and Winter (2002).


4 For example, in his account of sales practices, Cialdini (2001, page 208) reports: << Customers are often told that unless they make an immediate decision to buy, they will have to purchase the item at a higher price later or they will be unable to purchase it at all. [...] A home vacuum cleaner operation I infiltrated instructed its sales trainees to claim that, “I have so many other people to see that I have the time to visit a family only once. It’s company policy that even if you decide later that you want this machine, I can’t come back and sell it to you.”>>.

5 In the classical analysis of sequential bargaining, the incentive to reach an agreement can arise either from the parties’ (intrinsic) impatience or from an exogenous risk of negotiations breaking down. As shown by Binmore et al. (1986) and Sutton (1986), with classical preferences these two alternative interpretations are equivalent. Since reference-dependent preferences are a model of choice under risk and not of intertemporal choice — in this paper I focus on a model with exogenous breakdown risk without intertemporal discounting.

6 See Dato et al. (2015) for a comprehensive analysis regarding strategic interaction under expectation-based loss-aversion. To the best of my knowledge, my paper and theirs are the only ones to explicitly characterize a mixed-strategy equilibrium when players are expectations-based loss-averse.
buyer is loss-averse than when he is risk-neutral. Specifically, loss aversion makes the seller more likely to adopt a sequential screening strategy whereby in the first round she chooses a high price that only a high-valuation buyer would accept while in the second round she sets a lower price at which also a low-valuation buyer is willing to buy. Even though there is a significant difference in prices across periods, a high-valuation loss-averse buyer is willing to buy at a high price in the first period in order to avoid the feeling of loss resulting from a possible breakdown in future negotiations. Thus, compared to the risk-neutral case, in equilibrium the seller achieves higher profits and the overall probability of trade is higher. Moreover, I also show that the seller’s profits are non-monotone in breakdown risk. In particular, if the buyer is loss-averse the seller might increase her profits by reducing the probability of negotiations breakdown (at least over some range); such a prediction does not arise when the buyer is risk neutral as in this case the seller’s profits are strictly increasing in the breakdown probability. Hence, the predictions of the model are in line with the experimental evidence on sequential bargaining with buyer private information.

That buyer loss aversion makes it easier for a risk-neutral seller to adopt a screening strategy while also increasing the probability of trade is a novel result which stands in contrast with some other results that have appeared in the literature. For instance, in a static framework Hahn et al. (2015) show that expectations-based loss aversion limits the benefits of price discrimination for a risk-neutral seller and can even result in the optimality of a full pooling menu in situations where buyers with standard preferences would be separated via a screening menu. In particular, they suggest that given a (sufficiently) high level of loss aversion, a risk-neutral seller is more likely to shy away from screening in markets with a large population of consumers with low valuations. The reason behind their different result is that they assume the seller makes her offer before the buyer privately observes his valuation; that is, at the time of contracting there is symmetric information between the seller and the buyer because the latter is uncertain about his future demand. Relatedly, Benkert (2015) introduces expectations-based loss aversion into a model of bilateral trade à la Myerson and Satterthwaite (1983) and finds that, compared to risk-neutral benchmark, under loss aversion the problem of impossibility (of efficient trade) becomes even more severe. His model, however, is one of bilateral monopoly and two-sided incomplete information whereas in my model incomplete information is one-sided and the uniformed party has all the bargaining power. Hence, the opposite results about the effect of loss aversion on efficiency can be ascribed to the different assumptions with respect to information structure and bargaining power. Similarly, Herweg and Schmidt (2015) propose a theory of inefficient renegotiation based on loss aversion and show that loss aversion may prevent parties from achieving a materially efficient agreement or, worse, prevent renegotiation altogether. In their model, however, the parties’ reference point is given by the status quo at the time of renegotiation rather than their expectations. Hence, while previous contributions have focused on the role of loss aversion as friction and source of inefficiencies, my model shows that there is also a brighter side: in a dynamic environment loss aversion with an expectations-based reference point can actually be efficiency-enhancing.

In Section 3.1 I compare the predictions of the model with a loss-averse buyer with the ones that arise with a risk-averse buyer. While the predictions of the two models are qualitatively similar, I show via an example with CARA preferences that extremely high levels of risk aversion are necessary to replicate the results obtained under loss aversion. Hence, this calibration exercise suggests that loss aversion has a much stronger effect on the bargaining outcome than classical risk aversion — i.e., based on concavity/diminishing marginal utility of consumption — does.

Section 4 considers the case where the buyer’s valuation is drawn from a continuous distribution. The general qualitative results continue to hold in this more general case: loss aversion increases the seller’s profits as well as overall efficiency. Moreover, the model with continuously distributed types highlights an additional effect of loss aversion which does not appear in the simple two-type model. Besides increasing the seller’s profit and overall trade efficiency, loss aversion also redistributes surplus among the buyer’s types compared to the risk-neutral case. Indeed, I show that in equilibrium high-valuation buyers attain a lower utility level than under risk neutrality whereas the opposite holds for low-valuation buyers. The reason is that the first-period equilibrium price is higher with loss-averse buyers than with risk-neutral ones, whereas the opposite holds for the second-period price. Hence, some high-type buyers who buy in the first period under risk neutrality also buy in the first period under loss aversion, but at a much higher price. Conversely, those buyers who buy in the second periods under both risk neutrality and loss aversion end up paying a much lower price in the latter case. Furthermore, there is a positive measure of low-type buyers who never buy under risk neutrality — and hence in equilibrium obtain a surplus equal to zero — but instead buy in the second period and obtain a strictly positive surplus under loss aversion. This redistributive effect of loss aversion compared to risk neutrality arises directly from the seller’s heightened ability to sequentially screen the buyer’s type and is another novel implication of my model. In a complete information setup, Shalev (2002) shows instead that a loss-averse bargainer always achieves a lower equilibrium payoff than a risk-neutral bargainer. The difference between his result and mine stems from the fact that in his model there is only one type of buyer; therefore, there is no scope for the seller to engage in sequential screening.

My paper builds on the seminal contributions by Fudenberg and Tirole (1983) and Sobel and Takahashi (1983). Fudenberg and Tirole (1983) considered a two-period bargaining model where the traders’ private information is described by a two-point distribution. Sobel and Takahashi (1983) analyzed a similar model with one-sided uncertainty in which an uninformed seller makes all the offers and the buyer’s private valuation is drawn from a continuum of possible types. Both papers highlight that incomplete information is a major source of inefficiency in sequential negotiations. My model extends their work by showing how loss aversion on the buyer’s side increases the seller’s profits as well as overall efficiency.

There are two earlier papers introducing reference-dependent preferences into bargaining settings. Shalev (2002) extends Nash...
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(1950)'s classical axiomatic bargaining model to incorporate loss aversion and finds that increasing loss aversion for a player leads to worse outcomes for that player. This result emerges also in an alternating-offer game à la Rubinstein (1982) in which loss aversion modifies the notion of time preferences — in essence, loss aversion translates into higher impatience — and the reference point is the value of the outcome that could have been realized had the players reached an agreement in the previous period. Compte and Jehiel (2007) also introduce reference-dependent preferences into a bargaining game with complete information à la Rubinstein (1982). In their model the reference point is determined by the positions adopted by the players in prior bargaining phases. Although these positions are endogenously determined in equilibrium, the function that specifies how the reference point depends on past positions is exogenous. My model differs from those of Shalev (2002) and Compte and Jehiel (2007) on two main aspects. First, while their focus is on alternating-offer games with complete information, I consider a game with one-sided incomplete information in which the uniformed player has all the bargaining power. Second, following Köszegi and Rabin (2006), I assume that the loss-averse player's reference point is given by the expectations with which the player enters the negotiations. Hence, my formulation of the reference point is inherently "forward looking" whereas in their models the reference point is determined by former missed agreements and hence has a "backward looking" nature.

Section 5 concludes by recapping the results of the model and pointing out some of its limitations as well as possible avenues for future research. All proofs are relegated to Appendix A.

2. Model

2.1. Environment

A risk-neutral seller (she) and a loss-averse buyer (he) bargain over an indivisible object. The seller has a known production (or opportunity) cost of 0. The buyer has a private valuation for the good, unknown to the seller, equal to $v > 0$.

The buyer has expectations-based reference-dependent preferences as formulated by Köszegi and Rabin (2006). In this formulation, the buyer's utility function has two components. First, if he buys the good at price $p$, the buyer experiences consumption utility $v - p$. Consumption utility can be thought of as the classical notion of outcome-based utility. Second, the buyer also derives utility from the comparison of his actual consumption to a reference point given by his recent expectations (probabilistic beliefs).9

Slightly departing from the original model of Köszegi and Rabin (2006), I assume that the reference-dependent part of the buyer's utility is defined over his overall gains from trade (and not over each dimension of consumption utility). Hence, for a riskless consumption outcome $x \equiv v - p$ and a riskless reference point $x' \equiv v' - p'$, a consumer's total utility is given by

$$U(x|x') = x + \mu(x - x')$$

(1)

where

$$\mu(x - x') = \begin{cases} (x - x') & \text{if } x \geq x' \\ \lambda(x - x') & \text{if } x < x' \end{cases}$$

is gain-loss utility. The parameter $\lambda > 1$ is the coefficient of loss aversion, capturing the weight the buyer puts on losses relative to gains.11 The weight on gains is normalized to one. By positing a constant marginal utility from gains and a constant, but larger marginal disutility from losses, this formulation captures prospect theory's (Kahneman and Tversky, 1979; Tversky and Kahneman, 1991) loss aversion, but without its diminishing sensitivity.

Because in many situations expectations are stochastic, Köszegi and Rabin (2006) extend the utility function in (1) to allow for the reference point to be a probability distribution $F^r$. In this case a consumer's total utility from outcome $x$ can be written as

$$U(x|F^r) = x + \int_{x'} \mu(x - x')dF^r(x').$$

(2)

In words, when evaluating $x$ a consumer compares it with each possible outcome in the reference lottery. For example, if he expected to realize a surplus equal to either $10$ or $20$, then a surplus of $15$ feels like a loss of $5$ relative to the possibility of getting $20$, and, at the same time, like a gain of $5$ relative to the possibility of getting $10$. In addition, the weight on the loss (gain) in the overall experience is equal to the probability with which he had been expecting to get $20$ ($10$).

To complete this theory of consumer behavior, Köszegi and Rabin (2006) assume that beliefs must be consistent with rationality: a consumer correctly anticipates the implications of his plans and makes the best plan he knows he will carry through. Notice that any plan of behavior — which in my setting amounts simply to a price-contingent strategy of when to buy — induces some expectations. If, given these expectations, the consumer is not willing to follow the plan, then he could not have rationally formulated the plan in the first place. Hence, a credible plan must have the property that it is optimal given the expectations it generates.

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10 This is equivalent to assuming that the seller and the buyer are bargaining over how to split a pot of money, with only the buyer knowing how much money is in the pot. Such a formulation is appropriate if, for example, $v$ captures the object’s resale value; in this case, then, the buyer is only interested in his overall gains from trade.

11 One could also introduce a weight attached to gain-loss utility relative to intrinsic utility, $\eta \geq 0$. To simplify the analysis, I assume $\eta = 1$ which can be done without much loss, since this normalization does not qualitatively affect any of the results.
The timing of the game is as follows. At \( t = 0 \), the buyer learns his valuation and forms his plans which will in turn determine his reference point in the subsequent periods. At \( t = 1 \), the seller sets a price \( p_1 \) for the current period. The buyer either buys in the first period, in which case the game ends, or he rejects the offer. In the latter case, with probability \( 1 - \delta \) the game ends and with probability \( \delta \) the buyer and the seller meet again at \( t = 2 \) when the seller sets price \( p_2 \) and the buyer either buys or rejects the offer; in either case, the game then ends. I assume that the buyer’s reference point does not change between periods and that he breaks any indifference in favor of buying.\(^{12}\)

Let \( h^t \in H \) be a public history of rejected price offers.\(^{13}\) A seller’s strategy is a pricing distribution \( \sigma: H \to \Delta(\mathbb{R}_+) \), where \( \alpha_t(p) \) denotes the probability that the offer made by the seller at time \( t = 1, 2 \) is not greater than \( p \). A buyer’s strategy is denoted by \( \sigma: H \to [0, 1] \), where \( \sigma_t \) denotes the probability that the buyer accepts the seller’s price at time \( t = 1, 2 \). I interpret a buyer’s mixed strategy not as a randomization over buying/not buying to be carried out in a given period, but rather as a probability distribution over plans whose outcome is realized at time \( t = 0 \). Hence, if indifferent — at time \( t = 0 \) — between planning to buy at \( t = 1 \) and planning to buy at \( t = 2 \), the buyer randomizes between these plans, with the outcome of the randomization determining his reference point going forward. Of course the buyer can only randomize between plans that he knows he will follow through.

The solution concept is perfect Bayesian equilibrium (PBE) as defined by Fudenberg and Tirole (1991), with the additional requirement that the buyer’s strategy constitutes a (Preferred) Personal Equilibrium (Köszegi and Rabin, 2006). A perfect Bayesian equilibrium specifies history-dependent sequences of the seller’s price offers, the acceptance and rejection decisions of all buyer types, and the seller’s beliefs about the buyer’s types such that the strategies are best responses given the beliefs. The beliefs are updated from the strategies by applying Bayes’ rule whenever possible.

**2.2. Preliminaries: buyer’s behavior**

In this Section, I analyze the buyer’s behavior keeping the seller’s prices fixed. Notice that the buyer forms his plans at \( t = 0 \), before he learns the seller’s prices. Hence, the buyer needs to form some expectations about the prices that the seller will charge and, in equilibrium, these expectations must be correct. Let \( G \in \Delta(\mathbb{R}^+) \) denote the buyer’s expectations in period 0 about the prices he might face at \( t = 1, 2 \). Given his expectations about the seller’s choice of prices, the buyer plans to buy at \( t = 1 \) with probability \( \sigma_1 \) and at \( t = 2 \) with probability \( \sigma_2 \).

Let \( I_{G, \sigma} \) denote the distribution over final consumption outcomes induced jointly by \( G \) and \( \sigma = (\sigma_1, \sigma_2) \). In a Personal Equilibrium the behavior generating expectations must be optimal given the expectations:

**Definition 1.** \( \sigma \) is a Personal Equilibrium (PE) if

\[
U[\sigma I_{G, \sigma}] \geq U[\sigma' I_{G, \sigma}]
\]

for any \( \sigma' \neq \sigma \).

If there exist multiple credible plans, the buyer chooses the one that maximizes his expected utility from an ex-ante perspective:

**Definition 2.** \( \sigma \) is a Preferred Personal Equilibrium (PPE) if it is a PE and

\[
EU_{G, \sigma}[\sigma I_{G, \sigma}] \geq EU_{G, \sigma'}[\sigma' I_{G, \sigma}]
\]

for any \( \sigma' \) such that \( \sigma' \) is a PE.

It is easy to see that a buyer would never follow through the plan of buying at a price higher than his intrinsic valuation \( v \). Similarly, the plan to never buy constitutes a Personal Equilibrium only if both \( p_1 \) and \( p_2 \) are higher than \( v \).

Let \( p_1 \geq p_2 \) and consider now the plan to buy in the first period.\(^{14}\) Under this plan, the buyer’s reference point is to obtain an intrinsic payoff of \( v - p_1 \) with probability equal to one. If the buyer follows his plan and buys at \( t = 1 \), his utility is indeed \( v - p_1 \). If instead he deviates and decides to buy at \( t = 2 \), his utility is

\[
\delta (v - p_2) + \delta (p_1 - p_2) - \lambda (1 - \delta) (v - p_1).
\]

Expression (3) describes the buyer’s expectation at \( t = 1 \) of his realized utility at \( t = 2 \) if he deviates and decides to buy at \( t = 2 \) given that he expected to buy at \( t = 1 \). The term \( \delta (v - p_2) \) captures his expected consumption utility from buying in the second period at price \( p_2 \). The second term, \( \delta (p_1 - p_2) \), captures the gain the buyer expects to make with probability \( \delta \) in the second period by buying at a lower price. Finally, \( -\lambda (1 - \delta) (v - p_1) \) captures the fact that with probability \( 1 - \delta \) negotiations break down after the first period and the buyer ends up with nothing which feels like a loss compared to his original expectations of obtaining \( v - p_1 \). Therefore, the buyer will follow his original plan if

\[
v - p_1 \geq \delta (v - p_2) + \delta (p_1 - p_2) - \lambda (1 - \delta) (v - p_1) \Leftrightarrow p_1 \leq \frac{v(1 - \delta)(1 + \lambda) + 2\delta p_2}{1 + \delta + \lambda(1 - \delta)}.
\]

Hence, the plan to buy in the first period constitutes a Personal Equilibrium if

\(^{12}\) On the topic of how reference points evolve over time see Köszegi and Rabin (2009).

\(^{13}\) With this notation \( h^0 \in H \) is the empty history, which denotes the start of the game.

\(^{14}\) If \( p_2 > p_1 \) the buyer would never consider buying in the second period.
Consider now the plan to buy in the second period. Under this plan, the buyer’s reference point is stochastic as he expects to obtain a payoff of \( v - p_2 \) with probability \( \delta \) and a payoff equal to zero with probability \( 1 - \delta \). The buyer will not deviate from such a plan at \( t = 1 \) if

\[
P_1 \leq \min \left\{ v, \frac{v(1 - \delta) + 2\delta p_2}{1 + \delta + \lambda(1 - \delta)} \right\}. \tag{4}
\]

The expression on the left-hand-side of (5) represents the buyer’s (expected) utility at \( t = 1 \) from following his plan to buy at \( t = 2 \). Similarly, the expression on the right-hand-side of (5) describes the buyer’s utility if he deviates from his plan and buys at \( t = 1 \). The first term, \( v - p_1 \), is consumption utility. The term \(-\lambda \delta (p_1 - p_2)\) captures the loss from paying a higher price than expected since \( p_1 \geq p_2 \) and he expected to pay \( p_2 \) with probability \( \delta \). Finally, \((1 - \delta)(v - p_1)\) captures the fact that the seller expected negotiations to break down with probability \( 1 - \delta \) after the first period, in which case his surplus would have been 0; hence, in this case the buyer obtains a gain of \( v - p_1 \) compared to his original expectations of obtaining 0 with probability \( 1 - \delta \). Thus, the plan to buy in the second period constitutes a Personal Equilibrium if

\[
P_2 \leq \min \left\{ v, \frac{v(1 - \delta)}{\delta} \right\}. \tag{6}
\]

Combining (4) and (6), we have if \( p_1 > v \) and \( p_2 > v \) the buyer’s unique personal-equilibrium strategy is to never buy. For \( p_1 \leq v \) and \( p_2 \leq v \), the buyer’s unique personal-equilibrium strategy is to buy in the first period if \( p_1 < p_2, \delta + v(1 - \delta) \) and to buy in the second period if \( p_1 > \frac{v(1 - \delta)(1 + \delta + 2\delta p_2)}{1 + \delta + \lambda(1 - \delta)} \). For \( \frac{v(1 - \delta)(1 + \delta + 2\delta p_2)}{1 + \delta + \lambda(1 - \delta)} \geq p_1 \geq p_2, \delta + v(1 - \delta) \), instead, both plans are personal equilibria. Hence, in this case the buyer will select the plan that provides him with the highest expected utility at \( t = 0 \). If he plans to buy at \( t = 1 \) at price \( p_1 \), the buyer’s expected utility at \( t = 0 \) equals

\[
EU[(v - p_1)(p_1, p_2, 1, 0)] = v - p_1. \tag{7}
\]

Similarly, the expected utility of a buyer who plans to buy for sure at \( t = 2 \) at price \( p_2 \) is

\[
EU[(v - p_2)(p_1, p_2, 0, 1)] = \delta(v - p_2) + \delta(1 - \delta)(v - p_2) = \delta(1 - \delta)\lambda(v - p_2). \tag{8}
\]

The first term on the right-hand-side of (8) is standard expected consumption utility; the other terms capture expected gain-loss utility and are derived as follows. The buyer expects to get to the second period and buy at price \( p_2 \), obtaining surplus \((v - p_2)\), with probability \( \delta \); thus, if he buys in the second period he feels a gain of \((v - p_2 - 0)\) compared to the possibility of getting zero in the event of negotiations breaking down, which the buyer expected to happen with probability \( 1 - \delta \). Similarly, with probability \( 1 - \delta \) the buyer gets zero in the second period; hence, if negotiations break down the buyer feels a loss of \( \lambda(0 - (v - p_2)) \) compared to the possibility of getting \( v - p_2 \), which the buyer expected to happen with probability \( \delta \). We can re-write (8) as

\[
EU[(v - p_2)(p_1, p_2, 0, 1)] = \delta(v - p_2)[1 - (1 - \delta)(\lambda - 1)].
\]

Notice that expected gain-loss utility is always negative since \( \lambda > 1 \) and that risk neutrality is embedded in the model as a special case (for \( \lambda = 1 \)). Moreover, it is easy to see that planning to buy in period 2 at \( p_2 = v \) delivers zero expected utility whereas planning to buy at \( p_2 < v \) might deliver negative expected utility if \((1 - \delta)(\lambda - 1) > 1 \). Hence, the buyer’s utility at \( t = 2 \) might be non-monotone in the seller’s price. As this possibility looks rather unrealistic, I introduce the following assumption which, maintained for the remainder of the paper, guarantees that the buyer’s expected utility from planning to buy at \( t = 2 \) is positive whenever \( p_2 \leq v \):

**Assumption 1 (No dominance of gain-loss utility):** \( \lambda \leq 2 \).

In essence, Assumption 1 places an upper bound on \( \lambda \) and is sufficient to ensure that the buyer’s expected utility is monotonically increasing in his valuation.\(^{15}\) Comparing expressions (7) and (8) we see that, from an ex-ante perspective, the buyer prefers planning to buy in period 1 at price \( p_1 \) rather than planning to buy in period 2 at \( p_2 \) if

\[
p_1 \leq v(1 - \delta)[1 + \delta(\lambda - 1)] + p_2(1 - (1 - \delta)(\lambda - 1)]. \tag{9}
\]

Yet, for this plan to be a PE, it must be that once the buyer reaches period 1 expecting to buy at price \( p_1 \) he does not want to deviate in order to buy in period 2 at \( p_2 \); this is the case if

\[
v - p_1 \geq \delta(v - p_2) + \delta(p_1 - p_2) - \lambda(1 - \delta)(v - p_1) \Leftrightarrow p_1 \leq \frac{v(1 - \delta)(1 + \lambda) + 2\delta p_2}{1 + \lambda + \delta(1 - \delta)}. \tag{10}
\]

It is easy to verify that the right-hand-side of (10) is lower than the right-hand-side of (9) whenever \( v \geq p_2 \). In other words, the binding constraint for the plan to buy at \( t = 1 \) to be a PPE is that the buyer does not want to deviate from this plan at \( t = 1 \).\(^{16}\) Hence, if

---

\(^{15}\) Herweg et al. (2010) first introduced Assumption 1 and referred to it as “no dominance of gain-loss utility”. Several authors incorporating expectations-based loss-averse preferences into Bayesian games have imposed this assumption; see, for instance, Eisenhuth (2012), Lange and Ratan (2010) and Rosato (2014).

\(^{16}\) This follows because the buyer is willing to pay more ex-ante (at \( t = 0 \)) to avoid the risk of negotiations breaking down than he is ex-post (at \( t = 1 \)). See Köszegi and Rabin (2007).
both planning to buy at $t = 1$ and planning to buy at $t = 2$ are personal equilibria, the plan to buy at $t = 1$ is the preferred plan.

3. Binary types

Suppose the buyer’s private valuation for the good is equal to $v_H$ with probability $\pi \in (0, 1)$ and to $v_L \in (0, v_H)$ with probability $1 - \pi$. This is the setup considered by Fudenberg and Tirole (1983). In equilibrium, the game can reach the second period only if (at least) one type of buyer has planned to buy in this period with positive probability. It is easy to see that the highest price for which a buyer of type $\theta \in \{H, L\}$ would plan to buy at $t = 2$ and follow his plan is $p_2 = v_H$. Thus, if the game reaches the second period, the two relevant options for the seller are to either charge $p_2 = v_H$ in which case only a high-type buyer would buy, or to charge $p_2 = v_L$ in which case both types of buyer would buy. Let $\mu \equiv \Pr(v = v_H|t = 2)$ denote the beliefs about the likelihood of facing a buyer of type $H$ with which the seller enters period 2. Then we have

$$p_2 = \begin{cases} 
  v_H & \text{if } \mu > \frac{v_L}{v_H} \\
  v_L & \text{if } \mu < \frac{v_L}{v_H} \\
  \text{any randomization between } v_H \text{ and } v_L & \text{if } \mu = \frac{v_L}{v_H}
\end{cases}$$

To derive the seller’s optimal strategy in the first period, we must distinguish two cases for the seller’s prior beliefs, $\pi$. If $\pi < \frac{v_L}{v_H}$ then the seller will play $p_2 = v_L$ in the second period. Let this be the case of a “soft” (or pessimistic) seller. On the other hand, if $\pi \geq \frac{v_L}{v_H}$, then $\mu$ can be either above or below $\frac{v_L}{v_H}$ depending on the buyer’s first-period strategy. Let this be the case of a “tough” (or optimistic) seller. In what follows I shall analyze the two cases separately.

Suppose first that the seller is “soft”. Then, she will always set a price equal to $v_L$ in the second period. Furthermore, if she charges $p_1 = v_L$ then both types of buyer will buy in the first period so that she attains a profit of $v_L$. Alternatively, the seller can sequentially screen the buyer by charging a price $p_1 > v_L$ at which only the high-type buyer is willing to buy in the first period.

If he expects the seller to charge $p_1 = v_L$ in the second period, a high-type buyer prefers to buy in the first period if and only if

$$v_H - p_1 \geq \delta(v_H - v_L) + \delta(p_1 - v_H) - \delta(1 - \delta)(v_H - p_1) \iff p_1 \leq \frac{v_H(1 - \delta)(1 + \delta) - v_L}{1 + \delta + \delta(1 - \delta)} \equiv p^\text{soft}_1.$$

Therefore, with sequential screening the seller would charge $p^\text{soft}_1$ in the first period, expecting to sell only to the high-type buyer. Sequential screening provides the seller with a higher profit if and only if

$$\delta \pi^{\text{soft}} + (1 - \delta) \pi v_L \geq \delta \pi \Rightarrow \pi \geq \frac{v_L}{v_H(1 + \delta) - v_L}, \equiv \bar{\pi}.$$ 

Now, suppose that the seller is “tough”. Again, she has two possible options: sequential screening or selling only to the buyer of type $H$. Notice that $p^\text{soft}_1$ is the highest price at which a buyer of type $H$ would follow through the plan of buying in the first period when he anticipates that $p_2 = v_L$. What if $p_1 \in (p^\text{soft}_1, v_H]$? In this case, a buyer of type $H$ must mix in period 1. To see this, suppose he was planning to buy at $p_1 \in (p^\text{soft}_1, v_H]$. Then, $\mu$ would be equal to zero and $p_2 = v_L$. But then it would be better for a buyer of type $H$ not to buy in the first period. On the other hand, suppose $p_1 \in (p^\text{soft}_1, v_H]$ and he does not buy in period 1. Then, $\mu = \pi \geq \frac{v_L}{v_H}$ so that $p_2 = v_H$. But then it would be better for a buyer of type $H$ to buy in the first period. So, for $p_1 \in (p^\text{soft}_1, v_H]$ a buyer of type $H$ must mix in period 1. And he has to do it in such a way that at $t = 2$ the seller is exactly indifferent between charging $p_2 = v_L$ or $p_2 = v_H$. Let $\sigma_{L,H}$ be the probability that a type-$H$ buyer plans to buy in the first period; then, for the seller to be indifferent, we must have:

$$\mu = \frac{1 - \sigma_{L,H}}{\pi(1 - \sigma_{L,H}) + 1 - \pi} \Rightarrow \sigma_{L,H} = 1 - \frac{v_L(1 - \pi)}{\pi(v_H - v_L)}.$$

Notice that $\sigma_{L,H}$ does not depend on $p_1$. However, for a buyer of type $H$ to be willing to mix in period 0, he must be indifferent between planning to buy in period 1 and planning to buy in period 2. Let $\alpha_{L}$ be the probability that the seller charges $v_H$ and $1 - \alpha_{L}$ be the probability that she charges $v_L$ in period 2. If he plans to buy in period 1 at $p_1 \in (p^\text{soft}_1, v_H]$ and follows his plan, a high-type buyer achieves a utility level of $v_H - p_1$ whereas if he plans to buy in period 2 and follows his plan, his utility is equal to

$$(1 - \alpha_{L})\delta(v_H - v_L)[1 - (\lambda - 1)(\alpha_{L}\delta + 1 - \delta)].$$

The equilibrium mixed strategy for the seller is the $\alpha_{L}$ for which

$$v_H - p_1 = (1 - \alpha_{L})\delta(v_H - v_L)[1 - (\lambda - 1)(\alpha_{L}\delta + 1 - \delta)].$$

Notice that in the risk-neutral benchmark, the condition for a buyer of type $H$ to be willing to mix is

$v_H - p_1 = (1 - \alpha_{L})\delta(v_H - v_L).$
As the right-hand-side of (11) is less than \((1 - \alpha_2)\delta (v_H - v_l)\), it follows that if the buyer is loss-averse the seller will choose a lower \(\alpha_2\) compared to the risk-neutral case. Intuitively, because a loss-averse buyer dislikes the uncertainty ingrained in a mixed strategy, the seller must compensate him by charging the higher price less often. The solution to condition (11) is given by

\[
a^*_s(p_l) = \frac{1 + (\lambda - 1)(2\delta - 1)}{2\delta(\lambda - 1)} - \frac{\sqrt{(v_H - v_l)([1 + (\lambda - 1)^2][v_H - v_l] - 4(\lambda - 1)p_l + 2(\lambda - 1)(v_H + v_l))}}{2\delta(\lambda - 1)(v_H - v_l)}.
\]

Thus, a “tough” seller can either employ a sequentially screening strategy or, given that \(\sigma_{1t}\) is constant over \((p_l^{soft}, v_H)\), she could set \(p_l = v_H\) and randomize in the second period according to \(a^*_s(p_l)\) and attain an expected profit with probability \(p_l\), \(p_l^{soft}\), \(\pi\), and \(\pi v\). Hence, with loss aversion equilibrium prices are more dispersed across periods.

Finally, the overall probability of \((e_H L, l, \sigma_H L)\) is less than \((e_H L, l, \sigma_H L)\) with the sellervs.

The results are summarized in the following proposition.

**Proposition 1.** In the model with buyer with binary types there exist two thresholds values \(\tilde{\pi}\) and \(\bar{\pi}\), with \(\tilde{\pi} < \pi_H < \tilde{\pi}\) such that there is a unique perfect Bayesian equilibrium:

(i) if \(\pi \leq \tilde{\pi}\), each type of buyer plans to buy in the first period and the seller charges \(p_l = p_H = v_{Hl}\).

(ii) if \(\tilde{\pi} < \pi \leq \bar{\pi}\), the high-type buyer plans to buy in the first period, the low-type buyer plans to buy in the second period and the seller charges \(p_l = p_l^{soft}\) and \(p_l = v_H\).

(iii) if \(\pi > \bar{\pi}\), the high-type buyer mixes between the plan to buy at \(t = 1\) and the plan to buy at \(t = 2\) with probability \(\sigma_{1t}\) and \(1 - \sigma_{1t}\) respectively, the low-type buyer plans to never buy and the seller charges \(p_l = v_H\).

The next proposition compares the predictions of the model with a loss-averse buyer with the ones arising in a model with a risk-neutral buyer as in Fudenberg and Tirole (1983).

**Proposition 2.** Compared to the risk-neutral benchmark, the seller is more likely to adopt a sequential screening strategy when the buyer is loss-averse. Furthermore, loss aversion on the buyer’s side increases the seller’s profits as well as overall efficiency.

**Figs. 1 and 2** help to visualize the results in Proposition 2. First, it is easy to see that, compared to the risk-neutral case, if the buyer is loss-averse the seller is more likely to adopt a sequential screening strategy since \(\tilde{\pi} < \pi_H < \bar{\pi}\). Moreover, notice that in the model of Fudenberg and Tirole (1983) sequential screening is never profit-maximizing for a “soft” seller. In my model, instead, sequential screening might be profit-maximizing for a “soft” seller because loss aversion allows her to extract more rent from the high-type buyer without reducing the rent she extracts from the low-type buyer. It is also easy to check that loss aversion on the buyer’s side raises the seller’s expected profits as \(\frac{(v_H (1 - \delta)(1 + \lambda) + 2\delta v_l)}{1 + \delta + \lambda (1 - \delta)} > v_H (1 - \delta) + \delta v_l\). Hence, with loss aversion equilibrium prices are more dispersed across periods. Finally, the overall probability of (efficient) trade is higher under loss aversion as the range of prior beliefs for which the seller prefers to exclude the low-type buyer is reduced (\(\tilde{\pi} > \frac{v_H (1 - \delta)(1 + \lambda) + 2\delta v_l}{1 + \delta + \lambda (1 - \delta)}\)).

Table 1 describes the seller’s profit-maximizing strategy and her (expected) profits for various parameters’ configurations (the case \(\lambda = 1\) corresponds to the risk-neutral benchmark). Besides making sequential screening more likely in the sense of making it the seller’s profit-maximizing strategy for a larger measure of the seller’s prior beliefs \(\pi\), buyer loss aversion also generates a novel, interesting comparative-statics prediction with respect to \(\delta\). Indeed, it is easy to see from Table 1 that if the buyer is risk-neutral, the

\[
\begin{array}{cccc}
p_l = v_H & p_l = v_H (1 - \delta) + \delta v_l & p_l = v_H \\
p_l = v_H & p_l = v_H (1 - \delta) + \delta v_l & p_l = v_H
\end{array}
\]

**Fig. 1.** Equilibrium prices with a risk-neutral buyer.

\[
\begin{array}{cccc}
p_l = v_H & p_l = v_H (1 - \delta)(1 + \lambda) + 2\delta v_l & p_l = v_H \\
p_l = v_H & p_l = v_H (1 - \delta)(1 + \lambda) + 2\delta v_l & p_l = v_H
\end{array}
\]

**Fig. 2.** Equilibrium prices with a loss-averse buyer.
seller’s profits with a sequential screening strategy are (strictly) decreasing in \( \delta \). Yet, if the buyer is loss-averse the seller’s profits with a sequential screening strategy are not monotone in \( \delta \). Consider, for instance, the second row of Table 1, where \( \pi = 0.7 < 0.75 = \frac{3}{4} \).

If \( \lambda = 2 \), the seller prefers sequential screening over selling to both types of buyer in the first period if and only if \( \delta \geq 2/3 \) and her profits in this case are equal to

\[
\frac{3(28 - 5\delta - 3\delta^2)}{10(3 - \delta)}.
\]

It is straightforward to verify that the above expression is (strictly) increasing in \( \delta \) if \( \delta < 0.84 \) and decreasing otherwise. Similarly, it is to verify that for \( \pi = 0.5 > 8/30 = \frac{4}{15} \) and \( \lambda = 3/2 \), the seller’s profits with a sequential screening strategy are (strictly) increasing in \( \delta \) if \( \delta < 0.14 \) and decreasing otherwise. The following proposition shows that the non-monotonicity of the seller’s profits is a distinctive feature of buyer loss aversion.

**Proposition 3.** If the buyer is loss-averse, there exists a threshold \( \pi^* \) such that the seller’s profits with a sequential screening strategy are strictly increasing in \( \delta \) if and only if \( \pi < \pi^* \). In the risk-neutral benchmark, instead, the seller’s profits with a sequential screening strategy are always strictly decreasing in \( \delta \).

In the classical risk-neutral benchmark a higher \( \delta \) makes the option to wait for a lower price in the second period more attractive for a high-type buyer; hence, when it becomes more likely that the buyer and the seller will have a second opportunity to trade, a high-type buyer requires a significant price reduction to be willing to buy in the first period and, in turn, the seller needs to offer him a bigger discount in the first period. When the buyer is loss-averse, however, there is an additional effect of \( \delta \) via gain-loss utility which is non-monotone in \( \delta \). When \( \delta \) is relatively small, a further increase in the probability of reaching the second period exposes the buyer to greater uncertainty over his potential outcomes, thus reducing his utility. On the other hand, if \( \delta \) is relatively large to begin with, a further increase in the probability of reaching the second period reduces the buyer’s uncertainty over his potential outcomes and therefore increases his utility. Hence, the non-monotonicity of the seller’s profits in \( \delta \) when employing a sequential screening strategy is rooted in the non-monotonicity of the buyer’s gain-loss utility in \( \delta \). The result that under buyer loss aversion the seller’s profits from a sequential screening strategy can increase with \( \delta \) (up to some point) implies that the seller could benefit from increasing the probability of trading in the future, a prediction that could be tested with field data or in a laboratory experiment.

### 3.1. Loss aversion vs. risk aversion

I now compare the predictions of the model with a loss-averse buyer to the ones that arise with a risk-averse buyer. Let the buyer’s utility from outcome \( x \) be \( U(x) \) with \( U(0) = 0 \), \( U(\cdot) > 0 \) and \( U'(\cdot) < 0 \). What is the price \( p^*_H \) at which a high-type buyer is indifferent between buying in the first period or in the second period when he correctly anticipates that \( p_L = v_L \)? We have

\[
U(v_H - p^*_H) = \delta U(v_H - v_L) \Leftrightarrow p^*_H = v_H - U^{-1}(\delta U(v_H - v_L)).
\]

This price is higher than the one at which a risk-neutral high-type buyer would be indifferent if and only if

\[
U^{-1}(\delta U(v_H - v_L)) < \delta(v_H - v_L) \Leftrightarrow \delta U(v_H - v_L) < U(\delta(v_H - v_L))
\]

which follows from Jensen’s Inequality. Furthermore, we know that with risk neutrality on the buyer’s side a soft seller would never screen. What if the buyer is risk-averse? The seller will prefer the screening option over selling to both types in the first period if

---

**Table 1**

<table>
<thead>
<tr>
<th>( v_L/v_H )</th>
<th>( \pi )</th>
<th>( \lambda )</th>
<th>Strategy</th>
<th>Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/4</td>
<td>0.7 (soft)</td>
<td>1</td>
<td>price low</td>
<td>3</td>
</tr>
<tr>
<td>3/4</td>
<td>0.7 (soft)</td>
<td>2</td>
<td>price low if ( \delta &lt; 2/3 )</td>
<td>( \frac{3(28 - 5\delta - 3\delta^2)}{10(3 - \delta)} )</td>
</tr>
<tr>
<td>3/4</td>
<td>0.8 (tough)</td>
<td>1</td>
<td>screening if ( \delta \leq 12/13 )</td>
<td>( (16 - 6\delta)/5 )</td>
</tr>
<tr>
<td>3/4</td>
<td>0.8 (tough)</td>
<td>2</td>
<td>screening if ( \delta ) ( &lt; \delta \leq (47 - \sqrt{13})/18 )</td>
<td>( 3\frac{16 - 5\delta - \delta^2}{5(3 - \delta)} )</td>
</tr>
<tr>
<td>8/30</td>
<td>0.25 (soft)</td>
<td>1</td>
<td>price low</td>
<td>8</td>
</tr>
<tr>
<td>8/30</td>
<td>0.25 (soft)</td>
<td>1.5</td>
<td>price low if ( \delta &lt; 5/12 )</td>
<td>( \frac{5\pi + 0.75 - 126\delta^2}{213 - 3\delta} )</td>
</tr>
<tr>
<td>8/30</td>
<td>0.5 (tough)</td>
<td>1</td>
<td>screening if ( \delta \leq 60/137 )</td>
<td>15 - 7\delta</td>
</tr>
<tr>
<td>8/30</td>
<td>0.5 (tough)</td>
<td>1.5</td>
<td>screening if ( \delta \leq (152 - \sqrt{19429})/14 )</td>
<td>( \frac{225 - 120\delta^2}{5(3 - \delta)} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2(5 - \delta)</td>
</tr>
</tbody>
</table>

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must double. This is a very high degree of risk aversion. Indeed, reported estimates \( \delta \geq 0.4563 \) for which is a relatively conservative estimate for the loss aversion coe-

\[
\pi \left[ \delta \rho L - U^{-1}(\delta U(v_L - v_H)) \right] + (1 - \pi)\delta v_L > v_L \Leftrightarrow \pi > \frac{v_L(1 - \delta)}{\delta \rho L - U^{-1}(\delta U(v_H - v_L))} \equiv \tilde{\delta}^r.
\]

It is easy to see that \( \tilde{\delta}^r < \frac{v_L}{v_H} \), implying that sequential screening might be optimal for a “soft” seller if the buyer is risk-averse. Similarly, let \( \tilde{\delta}^r \) denote the threshold on the seller’s beliefs such that she is indifferent between using a sequential screening strategy and serving only the high-type buyer. It is easy to verify that \( \tilde{\delta}^r > \frac{v_L}{\delta \rho L + v_L} \), that is, a “tough” seller is more likely to use a sequential screening strategy if the buyer is risk-averse than if the buyer is risk-neutral. Thus risk aversion makes the same qualitative predictions as loss aversion. Yet, as the following example shows, the risk aversion model requires an extremely high degree of risk aversion to generate predictions of the same magnitude as the loss aversion model.\(^{18}\)

**Example 1 (Loss Aversion vs. CARA).** Suppose the buyer’s preferences are represented by \( U(x) = 1 - e^{-rx} \), with \( r > 0 \). In this case, we have

\[
p_1^* = v_H(1 - \delta) + \delta v_L + \frac{1}{r} \ln(\delta e^{-(1-\delta)(v_H-v_L)}) + (1 - \delta) e^{\delta(v_H-v_L)}.
\]

Comparing \( p_1^* \) with \( p_1^{\text{soft}} \) from Section 3 we get

\[
\lambda \leq \frac{(1 + \delta) \left[ \frac{1}{r} \ln(\delta e^{-(1-\delta)(v_H-v_L)}) + (1 - \delta) e^{\delta(v_H-v_L)} + \delta v_L + v_H(1 - \delta) \right] - 2\delta v_L - v_H(1 - \delta)}{(1 - \delta) \left[ \delta(v_H - v_L) - \frac{1}{r} \ln(\delta e^{-(1-\delta)(v_H-v_L)}) + (1 - \delta) e^{\delta(v_H-v_L)} \right]}
\]

Let \( \lambda = 1.5 \). Table 2 displays the values of \( \rho \) for which \( p_1^* \) is greater or equal to \( p_1^{\text{soft}} \) as a function of \( \delta \) and \( v_L/v_H \).\(^{19}\)

Consider, for instance, the first row of Table 2. For the risk aversion model to predict in the sequential-screening case a first-period price as high as the one of the loss aversion model, if the buyer has CARA utility his coefficient of absolute risk aversion must be at least 0.22816. Moreover, if \( v_L \) increases by 50%, \( r \) must double. This is a very high degree of risk aversion. Indeed, reported estimates for the coefficient of absolute risk aversion under the CARA specification are two orders of magnitude (or more) smaller.\(^{20}\)

**4. Continuum of types**

In this section, I analyze the case where the buyer’s types are distributed over an interval as in Sobel and Takahashi (1983). For simplicity, I assume that \( v \sim U[0, 1] \).

The following result from Fudenberg et al. (1985) considerably simplifies the study of the buyer behavior:

**Lemma 1 (Skimming or Cutoff -rule Property).** Suppose that the buyer accepts price \( p_1 \) at date \( t \) when his valuation is \( v \). Then he accepts price \( p_1 \) with probability one when he has valuation \( v > v' \).

**Lemma 1** implies that in equilibrium the seller’s posterior beliefs about the buyer’s valuation are a truncation of her prior beliefs. Therefore, for any price \( p_1 \) that the seller charges in the first period, if the buyer rejects the offer the seller’s posterior can be characterized by a unique value \( a^*(p_1) \). Intuitively, \( a^*(p_1) \) represents the highest possible valuation for the buyer, given that he did not buy in the first period at price \( p_1 \); i.e., the seller’s posterior beliefs in period 2 will be that \( v \sim [0, a^*(p_1)] \). The following proposition characterizes the unique equilibrium of the game.

\(^{18}\) I am grateful to one of the referees for suggesting the inclusion of this example.

\(^{19}\) Notice that \( \lambda = 1.5 \) is a relatively conservative estimate for the loss aversion coefficient as it implies that losses are weighted 50% more than gains. Most of the experimental studies on loss aversion provide estimates of \( \lambda \) between 2 and 3; see Gill and Prowse (2012), Karle et al. (2015) and Sprenger (2015).

\(^{20}\) Based on the choice of deductible of Israeli car drivers, Cohen and Einav (2007) estimate a median coefficient of absolute risk aversion from a CARA utility function of \( 3 \times 10^{-5} \). Using data from US consumers’ choices of deductibles for home insurance, Sydnow (2010) reports a median coefficient of absolute risk aversion from a CARA utility function of \( 2 \times 10^{-3} \).

---

### Table 2

Loss aversion vs. CARA.

<table>
<thead>
<tr>
<th>( v_L/v_H )</th>
<th>( \delta )</th>
<th>( p_1^* \geq p_1^{\text{soft}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/3</td>
<td>1/4</td>
<td>( r \geq 0.22816 )</td>
</tr>
<tr>
<td>1/2</td>
<td>1/4</td>
<td>( r \geq 0.45632 )</td>
</tr>
<tr>
<td>1/3</td>
<td>1/3</td>
<td>( r \geq 0.22681 )</td>
</tr>
<tr>
<td>1/2</td>
<td>1/3</td>
<td>( r \geq 0.45636 )</td>
</tr>
<tr>
<td>1/3</td>
<td>1/2</td>
<td>( r \geq 0.22407 )</td>
</tr>
<tr>
<td>1/2</td>
<td>1/2</td>
<td>( r \geq 0.44815 )</td>
</tr>
<tr>
<td>1/3</td>
<td>2/3</td>
<td>( r \geq 0.22126 )</td>
</tr>
<tr>
<td>1/2</td>
<td>2/3</td>
<td>( r \geq 0.44252 )</td>
</tr>
</tbody>
</table>
**Proposition 4.** In the model where types are uniformly distributed on \([0, 1]\) there exist a cutoff type \(a^*\) and prices \(p_1^*\) and \(p_2^*\) with \(a^* > p_1^* > p_2^*\) such that there is a unique perfect Bayesian equilibrium in which:

(i) buyers with \(v \in [a^*, 1]\) plan to buy in the first period at \(p_1^*\);

(ii) buyers with \(v \in (p_1^*, a^*)\) plan to buy in the second period at \(p_2^*\);

(iii) buyers with \(v \in (0, p_1^*)\) plan to never buy.

The explicit expressions of \(a^*, p_1^*\) and \(p_2^*\) are derived in the proof of **Proposition 4** in Appendix A. The next proposition compares the predictions of the model with loss aversion with the ones in the risk-neutral benchmark of Sobel and Takahashi (1983).

**Proposition 5.** The first-period price is higher with loss aversion than with risk neutrality; nonetheless, the probability of trade in the first period is also higher with loss aversion. Conversely, the second-period price is lower with loss aversion than with risk neutrality. Furthermore, loss aversion on the buyer’s side increases the seller’s profits as well as overall efficiency.

Let \(\tilde{p}_1, \tilde{a}\) and \(\tilde{p}_2\) denote the equilibrium first-period price, cutoff type and second-period price under risk neutrality, respectively. Fig. 3 helps to visualize the results in **Proposition 5**.

It is worth highlighting that even though with a loss-averse buyer the seller charges a higher first-period price compared to the risk-neutral case, the probability of trade in the first period is also higher when the buyer is loss-averse. On the other hand, the second-period price is lower with a loss-averse buyer, which implies that overall efficiency is higher under loss aversion as the measure of types which the seller prefers to exclude is reduced. Furthermore, it is easy to verify that \(p_1^* - p_2^* > \tilde{p}_1 - \tilde{p}_2\); that is, equilibrium prices are more dispersed when the buyer is loss-averse.

As the following proposition shows, when the buyer’s types are drawn from an interval loss aversion also has a redistributive effect.

**Proposition 6.** There exist a threshold \(\bar{v} \in (\tilde{p}_2, a^*)\) such that the equilibrium utility of loss-averse buyer with type \(v\) is lower than the equilibrium utility of a risk-neutral buyer of the same type if and only if \(v > \bar{v}\).

Since the second-period equilibrium price is higher under risk neutrality than under loss aversion, there is a positive measure of low-type buyers who would never trade in the former case but do in the latter. Moreover, those types who would buy in the second period under either risk neutrality or loss aversion are paying a lower price with loss aversion. So loss aversion “benefits” some buyers with lower types. On the other hand, those high-type buyers who would buy in the first period under either risk neutrality or loss aversion are paying a higher price with loss aversion. Finally, since the cutoff type with loss aversion is lower than the cutoff type with risk neutrality, there is also a positive measure of types who would buy in the second period if they were risk-neutral but buy in the first period under loss aversion. Hence, compared to the risk-neutral case, under loss aversion these buyers face a higher price but also a higher probability of trade. As shown in the proof of **Proposition 6** these types of buyer are also worse off under loss aversion. Hence, loss aversion “hurts” buyers with higher types. In particular, all the buyers who buy in the first period under loss aversion achieve a lower payoff than their risk-neutral counterparts.

### 5. Conclusion

Many factors might influence the positions people take when negotiating and, in order to proceed, each side must adjust her position throughout the negotiation. In this paper I have focused on the role of expectations-based loss aversion in sequential negotiations with one-sided asymmetric information and exogenous breakdown risk.

Loss aversion on the buyer’s side has two main consequences for the overall efficiency of the negotiations and the division of surplus. First, the seller’s equilibrium profit increases in the buyer’s degree of loss aversion. This occurs because the more loss-averse the buyer is, the more he is willing to pay at early stages to avoid the risk of negotiations breaking breakdown. More precisely, the seller can more effectively screen high-value buyers because these are the ones that have the biggest incentive to buy early to avoid the risk of breakdown. Second, there is an increase in the overall trade efficiency as loss aversion softens the rent-efficiency trade-off for the seller who can serve a large number of consumers at an earlier stage. If consumers’ valuations are continuously distributed, there is an additional effect: loss aversion benefits low-type buyers who buy in the second period as they do so at a price lower than the one they would face with classical (risk-neutral) preferences. Hence, loss aversion not only redistributes surplus from the high-type buyers to the seller, but it also increases the equilibrium payoff of some low-type buyers. I conclude the paper by discussing some of the model’s limitations, as well as possible directions for future research.

One natural possibility would be to move beyond two-period model and consider a more general model with a long (or infinite) horizon. If the buyer has expected-utility preferences, this would be a straightforward extension as expected utility has a convenient recursive structure so that in each period the buyer only needs to compare the utility from buying in the current period vs. the utility from buying in the next one. However, if the buyer is expectations-based loss-averse, such a recursive structure is not warranted. There are two main reasons for this. First, if the buyer is expectations-based loss-averse, the longer the horizon the more the possible...
deviations. Take for instance a 3-period model and consider a buyer who has planned to buy in the first period. When he enters the first period, the buyer has two possible deviations: waiting to buy in the second period or waiting to buy in the third (final) period. Each deviation is associated with a different lottery and a priori it is not clear which deviation would more attractive because expected gain-loss utility is not monotone in the probability of trade. Moreover, there is an additional issue that complicates the analysis with more than two periods. Consider again a 3-period model. Both the plan to buy in the second period and the plan to buy in the third period generate stochastic reference points. Hence, when choosing between these two plans at time 0, the buyer is basically comparing two lotteries. Take now a buyer who has planned to buy in the second period. When he considers whether to follow his plan in the second period, or deviating to buy in the third period, he is now in the second period and has observed the seller’s offer. Therefore, when considering whether to deviate from his plan or not, the buyer is comparing a sure outcome (buying now) with a lottery (buying next period). In other words, the comparison between the plan to buy in the second period and the plan to buy in the third period involves comparing two lotteries in period 0, but comparing a sure outcome to a lottery in period 2. This issue does not arise in the two-period model, where the comparison between the plan to buy in the first period and the plan to buy in the second period always entails a comparison between a sure outcome and a lottery.

Another potentially interesting extension would be to consider loss aversion on the seller’s side. While most of the recent literature in behavioral industrial organization has focused on the case of risk-neutral profit-maximizing sellers, some authors have also hinted at an effect of loss aversion on the seller’s side. For instance, using data from the real-estate market in the Boston metropolitan area, Genesove and Mayer (2001) find that home-owners whose houses’ market value has fallen below the original purchasing price tend to set higher selling prices and, as a result, their houses tend to stay longer on the market.21 Yet, Genesove and Mayer (2001) posit that a seller’s reference point is her previous purchasing price and hence their formulation of the reference point is inherently backward-looking. It is easy to see that if the seller is expectations-based loss-averse and the buyer is risk-neutral, the seller would also increase overall efficiency by making trade in the first period more likely if the buyer is risk-neutral. The case where both buyer and seller are loss-averse is more intricate and therefore left for future research.

Finally, in the model of this paper one party, the seller, had all the bargaining power. When the seller makes all the offers, the buyer’s types can be separated into two groups, those who accept the current offer, and those who reject the current offer in order to trade at more favorable terms in the future; hence, the equilibrium necessarily has a screening structure. In an alternating-offer game, however, there is also the opportunity for the buyer to signal his type, with higher buyer types trading off higher prices for a higher probability of acceptance. Indeed, in models of sequential bargaining with alternating offers and incomplete information many types of outcomes can be sustained in equilibrium. Yet, some of these equilibria seem rather implausible as they rely on the threat of adverse inferences following out-of-equilibrium offers.

Appendix A. Proofs

Proof of Proposition 1. In the text. □

Proof of Proposition 2. In order to prove the first statement, we need to show that \( \hat{\pi} < \frac{\nu_L}{\nu_H} \) and \( \hat{\pi} > \frac{\nu_L - \delta v}{\nu_H \delta v + \nu_L (1 - 2 \delta)} \). First, we have

\[
\frac{\nu_L}{\nu_H} \Rightarrow \frac{\nu_L \left[ 1 + \delta (1 - \delta) + \delta \right]}{\nu_H (1 + \delta) - \nu_L \delta (\delta - 1)} < \frac{\nu_L}{\nu_H} \Rightarrow \nu_L < \nu_H.
\]

Next, we have

\[
\frac{\nu_L}{\nu_H - \delta v} \Rightarrow \frac{\nu_L (\nu_H - \delta v)[1 + \delta + \delta (1 - \delta)]}{2 \nu_H \delta (\nu_H - \nu_L) + \nu_L (1 - \delta) \nu_H - \delta (\delta - 1) \nu_L} > \frac{\nu_L}{\nu_H} \frac{\nu_L - \delta v}{\nu_H \delta v + \nu_L (1 - 2 \delta)}
\]

\[
\Rightarrow \delta (1 - \delta)(\delta - 1)(\nu_H - \nu_L)^2 > 0.
\]

In order to show that loss aversion on the buyer’s side raises the seller’s expected profits it suffices to show that \( p_1^{soft} > \nu_H (1 - \delta) + \delta v \). We have

\[
p_1^{soft} > \nu_H (1 - \delta) + \delta v \Rightarrow \frac{\nu_H (1 - \delta)(1 + \delta) + 2 \nu_H \delta}{1 + \delta + \delta (1 - \delta)} > \nu_H (1 - \delta) + \delta v \Rightarrow \delta (1 - \delta)(\delta - 1)(\nu_H - \nu_L) > 0.
\]

Finally, overall efficiency is higher under loss aversion because \( \hat{\pi} > \frac{\nu_L}{\nu_H \delta v + \nu_L (1 - 2 \delta)} \); that is, the range of prior beliefs for which the seller prefers to exclude the low-type buyer is smaller under loss aversion than under risk neutrality. □

Proof of Proposition 3. The seller’s profits with a sequential screening strategy are equal to

\[
\pi p_1^{soft} + (1 - \pi) \delta v = \pi \left( \frac{\nu_H (1 - \delta)(1 + \delta) + 2 \nu_H \delta}{1 + \delta + \delta (1 - \delta)} \right) + (1 - \pi) \delta v.
\]

21 See also the related finance literature on the “disposition effect” (Odean, 1998; Barberis and Xiong, 2009).
Differentiating the above expression with respect to $\delta$ yields
\[
\frac{v_1(\lambda + \delta - \lambda \delta + 1)^2 - \pi(2v_1 - v_2 + 2\lambda v_2 + 2\delta v_2 + \lambda^2 v_2 + \delta^2 v_2 - 2\lambda^2 v_2 - 2\lambda^2 \delta v_2 + \lambda^2 \delta^2 v_2)}{(\lambda + \delta - \lambda \delta + 1)^2} > 0
\]
\[
\iff \pi < \frac{v_1(\lambda + \delta - \lambda \delta + 1)^2 - \pi(2v_1 - v_2 + 2\lambda v_2 + 2\delta v_2 + \lambda^2 v_2 + \delta^2 v_2 - 2\lambda^2 v_2 - 2\lambda^2 \delta v_2 + \lambda^2 \delta^2 v_2)}{2v_1(\lambda + 1) + v_2(\lambda - 1)(\lambda - 2\delta - \delta^2 - 2\lambda \delta + \lambda \delta^2 + 1)} \equiv \pi^*.
\]
Notice that for $\lambda = 1$, $\pi^*$ reduces to $\frac{v_1}{v_2}$. Hence, when the buyer is risk neutral, the seller’s profits with a sequential screening strategy are increasing in $\delta$ if and only if $\pi < \frac{v_1}{v_2}$; that is, if and only if the seller is soft. Yet, we know that with a risk-neutral buyer the seller would use a sequential screening strategy only when she is tough; i.e., $\pi \geq \frac{v_1}{v_2}$. Therefore, in the risk-neutral benchmark the seller’s profits are strictly decreasing in $\delta$. □

**Proof of Lemma 1.** See Fudenberg et al. (1985). □

**Proof of Proposition 4.** Let $\mathbb{E}[p_2|p_1]$ denote the price a buyer expects to face in period 2 if he rejects $p_1$ in period 1. Since in equilibrium the seller’s pricing strategy must be sequentially rational, it follows that $\mathbb{E}[p_2|p_1] < p_1$. The “marginal” type in period 1 is the buyer who is exactly indifferent between buying in the first and in the second period; that is, the buyer for which:
\[
\nu = v = p_1[1 + \delta + \lambda(1 - \delta)] - 2\delta \mathbb{E}[p_2|p_1] \equiv a^*(p_1).
\]

If the buyer does not buy in period 1 at $p_1$, the seller’s posterior beliefs in period 2 will be that $v \sim [0, a^*(p_1)]$. Therefore, following a rejection in period 1, in period 2 the seller will solve the following problem
\[
\max_{p_2} \left(1 - \frac{p_1}{a^*(p_1)}\right)p_2.
\]
It is easy to verify that the unique solution is equal to
\[
p^*_2(p_1) = \frac{a^*(p_1)}{2}.
\]
Substituting $\frac{a^*(p_1)}{2} = \mathbb{E}[p_2|p_1]$ into the definition of $a^*(p_1)$ and re-arranging yields
\[
a^*(p_1) = \frac{2p_1[1 + \delta + \lambda(1 - \delta)]}{2(1 - \delta)(1 + \lambda) + 2\delta}.
\]

In the first period, then, the seller solves the following problem
\[
\max_{p_1} U(p_1) = \Pr[v \geq a^*(p_1)]p_1 + \delta \Pr[p_2 < v < a^*(p_1)]p^*_2(p_1)
\]
\[
\quad = \left(1 - \frac{2p_1[1 + \delta + \lambda(1 - \delta)]}{2(1 - \delta)(1 + \lambda) + 2\delta}\right)p_1 + \delta \left(\frac{2p_1[1 + \delta + \lambda(1 - \delta)]}{2(1 - \delta)(1 + \lambda) + 2\delta}\right)^2.
\]
The first-order condition yields
\[
p^*_1 = \frac{2(1 - \delta)(1 + \lambda) + 2\delta}{2[1 + \delta + \lambda(1 - \delta)][4(1 - \delta)(1 + \lambda) + 4\delta - \delta[1 + \delta + \lambda(1 - \delta)]]}.
\]
Substituting $p^*_1$ back into the expressions for $a^*(p_1)$ and $p^*_2(p_1)$ we obtain
\[
a^* = \frac{2(1 - \delta)(1 + \lambda) + 2\delta}{4(1 - \delta)(1 + \lambda) + 4\delta - \delta[1 + \delta + \lambda(1 - \delta)]}
\]
and
\[
p^*_2 = \frac{2(1 - \delta)(1 + \lambda) + 2\delta}{8(1 - \delta)(1 + \lambda) + 8\delta - 2\delta[1 + \delta + \lambda(1 - \delta)]}.
\]
This concludes the proof. □

**Proof of Proposition 5.** Let $\hat{p}_1$, $\hat{\delta}$ and $\hat{\beta}_1$ denote the first-period price, marginal type and second-period price under risk neutrality, respectively. It is straightforward to verify that
\[
\hat{p}_1 = \left(1 - \frac{\delta}{2}\right)^2, \quad \hat{\delta} = \frac{1 - \delta}{2} \quad \text{and} \quad \hat{\beta}_1 = \frac{1 - \delta}{4(1 - \frac{3\delta}{4})}.
\]
Then, we have that $p^*_1 > \hat{p}_1$ if and only if

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Re-arranging condition (12) yields
\[
\frac{[2(1-\delta)(1+\lambda) + 2\delta]^2}{2[1 + \delta + \lambda(1-\delta)][4(1-\delta) + 4\delta - \delta[1 + \delta + \lambda(1-\delta)]]} > \frac{(1-\frac{\delta}{2})^2}{2\left(1 - \frac{3\delta}{4}\right)}.
\]

which is readily satisfied.

Next, we have that \(a^* < \bar{a}\) if and only if
\[
\frac{2(1-\delta)(1+\lambda) + 2\delta}{4(1-\delta) + 4\delta - \delta[1 + \delta + \lambda(1-\delta)]} < \frac{1-\frac{\delta}{2}}{2\left(1 - \frac{3\delta}{4}\right)}.
\]

Re-arranging (13) yields
\[
-\frac{1}{2}\delta^2(1-\delta)(\lambda - 1) < 0
\]
which is readily satisfied. Furthermore, it is easy to see that \(a^* < \bar{a}\) implies that \(p^*_2 < \bar{p}_2\) and hence that overall efficiency is higher under loss aversion since the measure of buyer’s types that gets excluded in equilibrium is reduced.

Finally, let \(U_S\) and \(\hat{U}_S\) denote the seller’s expected profits under loss aversion and risk neutrality, respectively. It is straightforward to check that
\[
\hat{U}_S = \left(\frac{2 - \delta}{4 - 3\delta}\right)^2 \left(\frac{\delta}{4} + 1 - \delta\right)
\]
and
\[
U_S = \left(\frac{H}{2H - 2K}\right)^2 \left(\frac{\delta}{4} + \frac{H - 2K}{2K}\right)
\]
where \(H \equiv 2(1-\delta)(1+\lambda) + 2\delta\) and \(K \equiv 1 + \delta + \lambda(1-\delta)\). Thus, we have that
\[
U_S > \hat{U}_S \Leftrightarrow 2(1-\delta)(1+\lambda) + 2\delta > [1 + \delta + \lambda(1-\delta)](2 - \delta) \Leftrightarrow \delta(1-\delta)(\lambda - 1) > 0
\]
which is readily satisfied. □

Proof of Proposition 6. In order to compare a consumer’s equilibrium utility with and without loss aversion, let’s divide the consumer’s types into five groups: (i) \([\hat{a}, 1]\), (ii) \((a^*, \bar{a})\), (iii) \([p^*_2, a^*]\), (iv) \([p^*_2, \bar{p}_2]\) and (v) \([0, p^*_2]\).

The equilibrium utility of a consumer whose type is in \([0, p^*_2]\) is zero both under risk neutrality and loss aversion as he does not buy in either case. A consumer whose type is in \([p^*_2, \bar{p}_2]\) is strictly better off under loss aversion than under risk neutrality as in the latter case he does not get to buy whereas in the former he does. Conversely, a consumer whose type lies in \([\hat{a}, 1]\) is worse off under loss aversion than risk neutrality since he buys in the first period in both cases, but he pays a higher price with loss aversion.

Next, consider a consumer with type in \((a^*, \bar{a})\). Such a consumer buys in the first period under loss aversion and in the second period under risk neutrality. Hence, in order to establish whether the consumer is worse off under loss aversion, we need to compare the cost from having to pay a higher price vis-a-vis the benefit from consuming with a higher probability. Hence, a consumer with \(v \in (a^*, \bar{a})\) is better off under loss aversion if and only if
\[
v - p^*_2 > \delta(v - \bar{p}_2) \Leftrightarrow v > \frac{p^*_2 - \bar{p}_2}{1 - \delta}.
\]

Recall that \(\bar{a}\) is the type who, in the risk-neutral case, is exactly indifferent between buying in the first or in the second period; that is:
\[
\bar{a} - \bar{p}_2 = \delta(\bar{a} - \bar{p}_2) \Leftrightarrow \bar{a} = \frac{\bar{p}_1 - \bar{p}_2}{1 - \delta}.
\]

It is easy to see that \(p^*_2 - \bar{p}_2 > \bar{a}\) since \(p^*_2 > \bar{p}_2\). Therefore, a consumer with \(v \in (a^*, \bar{a})\) is worse off under loss aversion.

Finally, consider a consumer whose type is in \([\bar{p}_2, a^*]\). Such a consumer buys in the second period under both loss aversion and risk neutrality. Yet, with loss aversion he pays a lower price but he also suffers the costs associated with the risk of negotiations breaking down. Hence, a consumer with \(v \in [\bar{p}_2, a^*]\) is worse off under loss aversion if and only if
\[
\delta(v - \bar{p}_2) > \delta(v - p^*_2)[1 - (1 - \delta)\eta(\lambda - 1)].
\]

It is easy to see that if \(v = \bar{p}_2\) the consumer is better off under loss aversion as in this case he makes a strictly positive surplus.
where with risk neutrality his surplus would be equal to zero. On the other hand, since $a^*$ is the type of buyer who is indifferent between buying in the first or in the second period with loss aversion, for $v = a^*$ we have

$$v - p_1^* = \delta (v - p_1^* + \delta (p_1^* - p_2^*)) - \Delta (1 - \delta) (v - p_1^*) > \delta (v - p_1^*) [1 - (1 - \delta) (1 - \delta)]$$

where the inequality follows because if in the first period the buyer is indifferent between following his plan to buy in the first period or deviating to buy in the second period, then from an ex-ante perspective he strictly prefers planning to buy in the first period over planning to buy in the second period; see conditions (9) and (10). Yet, from our previous analysis, we also know that $v - p_1^* < \delta (v - p_2^*)$ if $v < \tilde{a}$. Hence, since $a^* < \tilde{a}$, if $v = a^*$ the consumer is worse off under loss aversion. Since a consumer’s equilibrium utility is continuous and strictly increasing in his type, it follows that there exists a value $\tilde{v} \in (p_2, a^*)$ such that the consumer is worse off under loss aversion if and only if $v > \tilde{v}$. □

References


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